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THE DOUBLING METHOD APPLIED TO MULTIPLE SCATTERING OF POLARIZED LIGHT

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Abstract—The doubling method is used for computations of multiple scattering with full account of polarization. Several checks on the accuracy of the method are indicated. Graphs are presented for the polarization of sunlight reflected by the visible disk of Venus for Mie scattering phase matrices. The results show that photons scattered two or more times still have a high degree of polarization which must be included in a complete interpretation of observations.

WE USED the doubling method for multiple scattering computations including polarization. The azimuth dependence was handled by making expansions in Fourier series. The lowest terms in the Fourier series were computed by means of the doubling method with a starting layer thin enough that two orders of scattering were sufficient. The higher terms were obtained, with no loss of accuracy, by calculating only two orders of scattering; we previously found that this was satisfactory for scattering without polarization and this point was discussed earlier in this symposium by VAN DE HULST.[11]

Several comparisons were made between our numbers and those of independent researchers. In the case of conservative Rayleigh scattering a very demanding check was obtained by continuing the doubling until a layer of practically semi-infinite optical thickness was reached. In this way, at an optical thickness $\sim 10^6$, we obtained agreement to about five decimals with the results published by CHANDRASEKHAR.[1] In the case of Rayleigh scattering with absorption we obtained agreement to four decimals with the numbers computed by ABHYANKAR and FYMAT.[2] Checks were also made for non-Rayleigh scattering. For example, the reflection and transmission were computed for a layer consisting of identical spheres of size parameter $X = 5$; the results were compared with those obtained by DAVE[3] and the agreement was usually to two significant figures. All of these authors used methods which were quite different from ours and in each case the agreement was to the full accuracy expected by those authors. From these comparisons, as well as other checks, we have found that we can obtain accuracies of at least three to four figures for intensities and 0.1 per cent for polarizations in reasonable computing times.

It is one thing to obtain checks on a computing method but quite another to prove its worth on realistic problems. Here the "Excalibur", for any method of computing the multiple scattering of polarized light, lies in the detailed information about the planet Venus which could be extracted from long available polarization observations. LYOT,[4]

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as early as 1922, made accurate observations of the linear polarization of sunlight reflected by Venus; the observations have been extended by COFFEEN, GEHRELS and DOLLFUS[5,6] to several wavelengths from the u.v. to the near i.r. The best interpretation to date of the observations is that made by COFFEEN[7] on the basis of calculations for single scattering by spherical particles: Coffeen concluded that the cloud particles on Venus had an average radius of $\sim 1.25\mu$ and a refractive index in the range $1.43 \leq n_r \leq 1.55$. However, because of possible unknown systematic effects of multiple scattering on the polarization, his interpretations are tentative, and, in any case, exact computations of the multiple scattering can yield more precise information on the atmosphere and clouds of Venus.

We have made some calculations relevant to the Venus problem. Figures 1 and 2 give the computed linear polarization of the reflected sunlight received from the visible disk of Venus at all phase angles. These calculations are for particular models of the Venus clouds: Mie scattering phase matrices were used with the size distribution of particles following the gamma function used by DEIRMENDJIAN[8] and HANSEN and POLLACK.[9] The results then depend on the mean scattering radius which is the mean particle radius for the size distribution weighted by the scattering cross section. The atmosphere was taken as being locally plane parallel and both reflection from the planetary surface and Rayleigh scattering by the gas molecules were neglected. The optical thickness and single scattering albedo were chosen such that the resulting Bond albedos were 80 and 90 per cent at $\lambda = 0{\cdot}55\ \mu$ and $0{\cdot}99\ \mu$, respectively. The results presented here are illustrative only and inadequate for a derivation of detailed atmospheric conditions on Venus.

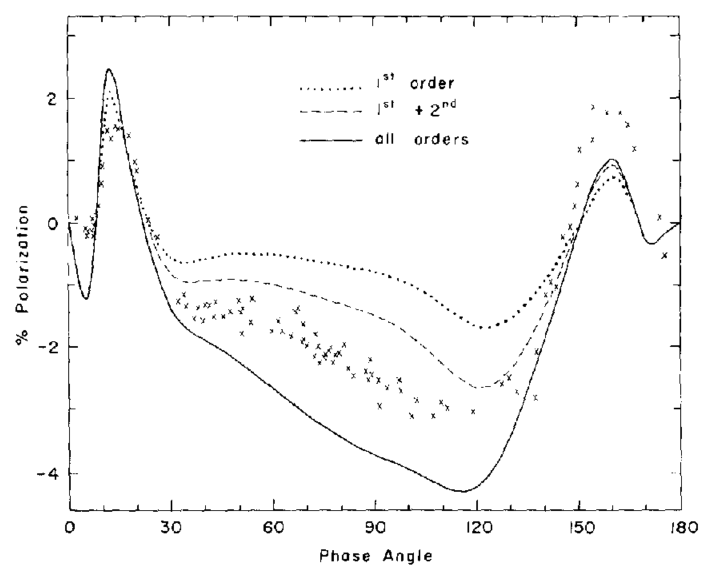

FIG. 1. Variation of the degree of polarization with phase angle for the visible disk of the planet Venus. The crosses represent the visual observations of LYOT.[4] The curves were obtained from calculations for spheres with a refractive index 1·5 and a mean scattering radius of $1{\cdot}2\ \mu$. A size distribution following Deirmendjian's cloud model was assumed. The wavelength for the computations was $0{\cdot}55\ \mu$, the albedo of single scattering 0·99738, and the optical thickness 128. 1st order: Q from single scattering, I from all orders of scattering. 1st and 2nd: Q from single and secondary scattering, I from all orders. All orders: Q and I from all orders of scattering.

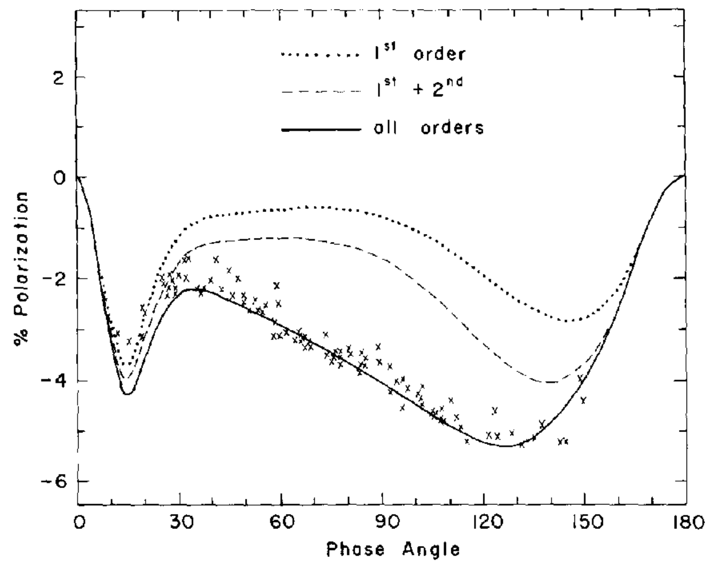

FIG. 2. Variation of the degree of polarization with phase angle for the visible disk of the planet Venus at $\lambda = 0{\cdot}99\ \mu$. The crosses represent observations of COFFEEN and GEHRELS.[5] The curves were obtained from calculations for spheres with a refractive index 1·45 and a mean scattering radius of 1 μ. A size distribution following Deirmendjian's cloud model was assumed. The computations were made for an albedo of single scattering 0·99932 and an optical thickness 128. The curves have the same meaning as in Fig. 1.

Figures 1 and 2 include curves labeled "1st order"; these represent the polarization which would occur if photons scattered more than once were unpolarized. Thus this approximation consists of using all orders of scattering for the intensity but only single scattering for the other Stokes parameters; in practice the intensity for this approximation could be computed by using the simple case in which polarization is neglected or it could be taken from observations. The "first order" approximation was suggested by VAN DE HULST[10] and it represents a considerable improvement of the approximation of LYOT,[4] which consisted of applying a constant reduction factor at all phase angles to account for multiple scattering. However, it is obvious from both figures that photons scattered two, three, or more times are often still highly polarized. Therefore, in a derivation of detailed atmospheric characteristics on Venus it will be necessary to account for the polarization of several orders of scattering.

COFFEEN[7] based his conclusions about Venus partly on the locations of the neutral points, i.e. the zero points in the polarization vs. phase angle curve; his most crucial assumption was that multiple scattering would not move these curves much. Figure 1 indicates that his basic assumption was very good for a polarization curve such as that occurring for Venus at visual wavelengths. For the case shown in Fig. 2 there are no nontrivial neutral points; for a similar phase matrix, but with the first order curve extending just into the positive polarization domain, multiple scattering can cause the zero points to move greatly or even disappear. Nevertheless, our results basically bear out the validity of COFFEEN'S assumption. A multiple scattering interpretation of the observations can greatly narrow the limits on the cloud particle properties and provide new information, but it will probably not yield a refractive index much outside the range specified by COFFEEN.

Figures 1 and 2 also indicate that multiple scattering does not smooth out sharp angular features in the polarization, even though orders of scattering higher than the first contribute significantly to the polarization. This is distinctly different from the case for intensities[9] and this difference in part accounts for the higher information content available in polarization observations. The sharpness of the angular variations in the polarization increases markedly for larger particles or shorter wavelengths; hence it is obvious that approximate computing methods giving results averaged over intervals of 5° or 10° will not be capable of extracting the full information available in observations.

More detailed descriptions of the doubling method for polarized light including calculations for different applications can be found elsewhere.[12,13] A preliminary interpretation of the calculations for Venus has been published[14] and further work on Venus is in progress.

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